Title: Discover Logarithms

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Topics: Logarithms and their properties. Arithmetic and geometric series.

Connection to Core Curriculum: None found

Overview: We will use arithmetic and geometric series to help students discover the properties of logarithms.

Objectives: Students learn the addition, subtraction, multiplication, and division properties of logarithms.

Materials Needed: A board and students to have a paper and pencil.

Web Reference:
http://books.google.com/books?id=s7JoNDSA9zAC&pg=PA49&dq=Napier%27s+logarithms+adapted+for+today%27s+classroom&hl=en&sa=X&ei=RhiUUsTGEMfgoASVsoKABg&ved=0CC0Q6AEwAA#v=onepage&q&f=false

Lesson plan from the a book entitled *Learn from the Masters!*, edited by Fank Swetz

http://www.maa.org/publications/periodicals/convergence/logarithms-the-early-history-of-a-familiar-function-john-napier-introduces-logarithms

Logarithms: The Early History of a Familiar Function
Author(s): Kathleen M. Clark (The Florida State University) and Clemency Montelle (University of Canterbury)
DOI:10.4169/loci003495

Activity Plan:

1. Set up an arbitrary arithmetic sequence beginning with 0 having a constant difference of 1. Place below this a geometric sequence beginning with 1 having a constant ratio of r>1.

0	1	2	3	4	5	6	7	…
1	r	r^2	r^3	r^4	r^5	r^6	r^7	…

2. Have students conjecture how these two sets of numbers relate on their own paper. Then, show them how we can relate these two sets of numbers using the following notation: $f(n) = r^n$ and $g(r^n)=n$. This should be similar to many of the student's conjectures, just different notation.

3. Ask students to multiply values in the second set of numbers together. For example, r^3 times r^2 would be r^5. Now, how do the numbers on the first row relate when we are doing these multiplications? 3 corresponds to r^3 and 2 corresponds to r^2 and 5 corresponds to r^5. Have them do a few examples until the students see that we add the values in the first row to obtain the values in the second row.

4. Lead them through a similar example to see that a division in the second row corresponds to subtraction in the first row.

5. Next, lead them through a similar exercise but raise the values in the second row to an additional power. For example $(r^3)^4$ would be r^{12}. 3 corresponds to r^3 so what is the relationship this is representing? Raising values in the second row by a constant corresponds to multiplying by that constant in the first row.
6. Now, ask what would happen to the second row if you were to take a logarithm base r of each of the entries. Students should quickly realize that this would produce the first row of numbers.

   So the first row corresponds to the logarithm while the second row refers to the argument you are taking the log of.
7. With this in mind, have them write down each of the logarithm properties that they discovered earlier in the lesson using logarithmic notation.

   I.e. log(a)+log(b) = log(a*b), log(a)-lob(b) = lob(a/b) and so forth.

8. Read about Napier's logarithms and have students write a few paragraphs about how what we did in class relates to how Napier discovered logarithms. (Go to http://www.maa.org/publications/periodicals/convergence/logarithms-the-early-history-of-a-familiar-function-john-napier-introduces-logarithms for a good history on Napier's discovery process.)